Introduction to Quad topological spaces (4-tuple topology).

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Abstract

In this paper, we introduce a new concept Quad topological spaces (4-tuple topology) and defined new types of closed sets q-closed, q-b closed and q-b τ closed sets on new topology.

Keywords: Quad topology, q-open set, q-b open set, q-b τ open set

1 Introduction

Recently the topological structures had a lot of applications in many real life situations. Starting from single topology it extended to bitopology and tritopology with usual definitions. The concept of a bitopological space was first introduced by Kelly [1] and extention to tri-topological spaces was first initiated by Martin M. Kovar [2] in 2000, where a non empty set X with three topologies is called a tri-topological space. In this paper we introduce the concept on topological structures with four topologies, quad topology (4-tuple topology) and defined new types of closed (open) sets.

2 New concept and Definition

Definition 1 (Quad Topology) *Let X be a nonempty set and τ_1, τ_2, τ_3 and τ_4 are general topology on X. Then a subset A of space X is said to be quad-open (q-open) set if $A \in \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$ and complement is said to be q-closed set and the set X with four topologies called $(X, \tau_1, \tau_2, \tau_3, \tau_4)$* ***Quad topology*** *(q-topology). Obviously by definition of q-open sets it satisfies all the axioms of topology.*

Ex.1 Let $X = \{a,b,c,d\}, \tau_1 = \{\varnothing, X, \{a\}\}, \tau_2 = \{\varnothing, X, \{b\}\}, \tau_3 = \{\varnothing, X, \{c\}\}, \tau_4 = \{\varnothing, X, \{d\}\}$, then the sets $\varnothing$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$ are q-open sets and $\varnothing, X, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,c,d\}$ are all q-closed sets in $(X, \tau_1, \tau_2, \tau_3, \tau_4)$.

Definition 2 *A subset A of a q-topological space $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ is called q-neighborhood of a point $x \in X$ if and only if there exists an q-open set U such that $x \subset U \subset A$.*

Ex.2 Let $X = \{a,b,c,d\}, \tau_1 = \{\varnothing, X, \{a\}\}$, $\tau_2 = \{\varnothing, X, \{a,b\}\}$, $\tau_3 = \{\varnothing, X, \{c\}\}$,and $\tau_4 = \{\varnothing, X, \{d\}\}$ since the set $\{a\}$ is q-open and $a \in \{a\} \subseteq \{a,b\}$, then $\{a,b\}$ is q-neighborhood of a.

Remark.1 *We will denote the q-interior (resp.q-closure) of any subset, say A of X by q-int A (resp. q-clA), where q-int A is the union of all q-open sets contained in A, and q-clA is the intersection of all q-closed sets containing A.*

Definition 3 *A subset A of a space X is said to be q-b open set [2] if $A \subset$ q-cl(q-int A) $\cup\, q-int(q-clA)$.*

Remark.2 *1. The complement of q-b open set is called q-b closed set. Thus $A \subset X$ is q-b closed if and only if q-cl(q-int A)$\cap$ q-int(q-clA)$\subset$ A. The intersection of all q-b closed sets of X containing a subset A of X is called q-b closure of A and is denoted by q-clb(A).*

2. The intersection of all q-b closed sets of X containing a subset A of X is called q-b closure of A and is denoted by q-$cl_b(A)$. Analogously the q-b interior of A is the union of all q-b open sets contained in A denoted by q-$int_b(A)$.

Definition 4 *A subset A of a quad topological space X is called to be q-bτ-closed if q-$cl_b(A) \subset U$ whenever $A \subset U$ and U is q-open.*

Remark.3 *1.The complement of q-bτ-closed is q-bτ-open.*

2. The intersection of all q-bτ-closed sets of X containing a subset A of X is called q-bτ-closure of A and is denoted by q-$cl_{b\tau}(A)$. Analogously the q-bτ-interior of A is the union of all q-bτ-open sets contained in A denoted by q-$int_{b\tau}(A)$.

Theorem 1.

The relationships between the concepts q-closed set, q-b closed set and q-bτ closed sets summarized as below:

q-closed$\Rightarrow$ q-b closed $\Rightarrow$ q-bτ closed.

1. First we prove the statement q-closed$\Rightarrow$q-b closed.

Let $A \subseteq X$ be q-closed set. Hence A^c (complement of A) is q-open set. Since $A^c \subset$ q-cl (A^c),

q-$int(A^c) \subset$ q-int(q-cl A^c). But q-int A$\subset$ for any subset A, hence $A^c \subset$ q-int(q-cl A^c), and $A^c \subset$ q-int(q-cl A^c)$\cup$ q-cl(q-int A^c), hence A^c is q-b open set, hence A is q-b closed set.

2. Next we have to prove q-b closed$\Rightarrow$ q-bτ closed. Let A be a q-b closed subset of X and let $A \subseteq U$, where U is q-open. Since A is q-b closed set, hence q-int(q-cl(A))$\cap$ q-cl(q-int(A))

$\subset A$. But $A \subset U$, hence q-int(qcl(A)) $\cap$ q-cl(q-int(A)) $\subset U$. Also q-$cl_b(A)$ is the smallest q-b closed set containing A, so

q-$cl_b(A) = A \cup$ (q-int(q-cl(A)) $\cap$ q-cl(q-int(A)))
$\subset A \cup U$
$\subset U$.
Hence A is q-bτ closed set.

The inverse of the above theorem need not be true. We will illustrate this by following simple examples.

Ex.3 Every q-b closed set is not a q-closed set.

Let $X = \{a, b, c\}$,
$\tau_1 = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\}, \tau_2 = \{\varnothing, X, \{b\}\}, \tau_3 = \{\varnothing, X, \{a\}\}, \tau_4 = \{\varnothing, X, \{a, b\}\}$, then the sets $\varnothing, X, \{c\}, \{b, c\}\}, \{a, c\}$ are q-closed sets and the sets $\varnothing, X, \{a, c\}, \{b, c\}, \{a,\}, \{b\}, \{c\}$ are q-b closed sets. So $\{a,\}, \{b\}$ are q-b closed but not q-closed sets.

Ex.4 Every q-bτ closed set not a q-b closed set.
In the above example, the set $\{a, b\}$ is q-bτ closed set but it is not a q-b closed set.

3 Future work

1. Introduce all concepts (compactness,connectedness etc) of general topology.
2. Investigation of relation between quad topology and general topology
3. Study on separation axioms using newly defined open sets.
4. To find applications to digital quad topology

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